Electronic Supplementary Material

Deriving the Optimal Decision Rule

Suppose that the environment is in state 1, and let $\Pr(1|x, y)$ be the probability that the environment is in state 1 given that the individual observes two cues, x and y. Then due to the symmetry of the model, the optimal decision rule is to adopt behavior 1 if $\Pr(1|x, y) >$ ½, otherwise adopt behavior 2.

The first step in calculating $\Pr(1|x, y)$ is to calculate the joint probability that the environment is in state 1, $\Pr(1)$ and that the individual observes x and y, $\Pr(1, x, y)$. Since $\Pr(x|1)$ and $\Pr(y|1)$ are independent,

$$\Pr(x, y|1) = \Pr(x|1)\Pr(y|1)$$

and thus

$$\Pr(1, x, y) = \Pr(x, y|1)\Pr(1) = \Pr(x|1)\Pr(y|1)\Pr(1)$$

Using Bayes law to calculate $\Pr(1|x, y)$

$$\Pr(1|x, y) = \frac{\Pr(1, x, y)}{\Pr(x, y)}$$

where $\Pr(x, y) = \Pr(x, y|1)\Pr(1) + \Pr(x, y|2)\Pr(2)$. Thus the expression above becomes

$$\Pr(1|x, y) = \frac{\Pr(x|1)\ \Pr(y|1)\Pr(1)}{\Pr(x|1)\ \Pr(y|1)\Pr(1) + \Pr(x|2)\ \Pr(y|2)\Pr(2)}$$

Since the environment is equally likely to be in either state, Pr(1) = Pr(2)

$$\Pr(1|x, y) = \frac{\Pr(x|1)\ \Pr(y|1)}{\Pr(x|1)\ \Pr(y|1) + \Pr(x|2)\ \Pr(y|2)}$$

It is useful to rewrite this expression as

$$\Pr(1|x, y) = \frac{\frac{\Pr(y|1)}{\Pr(y|2)}}{\frac{\Pr(y|1)}{\Pr(y|2)} + \frac{\Pr(x|2)}{\Pr(x|1)}}$$

Thus, the probability that the environment is in state 1 given the cues will be greater than ½ when

$$\frac{\Pr(y|1)}{\Pr(y|2)} > \frac{\Pr(x|2)}{\Pr(x|1)}$$

So far the derivation is completely general. Now let us assume that each individual learns by observing an environmental cue that can take on a range of values. The environmental cue is a normally distributed random variable with mean μ and variance v when the environment is in state 1 and with mean -μ and variance v when the environment is in state 2. Let us further assume that models are sampled at random from the previous generation so that $\Pr(y|1)$ is a binomial distribution with parameters p and n and $\Pr(y|2)$ is a binomial with parameters$(1-p)$ and n. This means that individuals adopt behavior 1 if

$$\frac{\frac{n!}{y!\,(n-y)!}p^{y}\,(1-p)^{n-y}}{\frac{n!}{y!\,(n-y)!}p^{n-y}(1-p)^{y}} > \frac{e^{-\frac{(x+\mu)^2}{2v}}}{e^{-\frac{(x-\mu)^2}{2v}}}$$

where x is the value of the cue observed by the individual. The expression can be simplified to

$$\left(\frac{p}{1-p}\right)^{2y-n} > e^{-\frac{(x^2+2x\mu+\mu^2)-(x^2-2x\mu+\mu^2)}{2v}}$$

Or

$$\left(\frac{p}{1-p}\right)^{2y-n} > e^{-\frac{2x\mu}{v}}$$

Since the logarithm function is monotonic, we can take the logarithm of both sides of the inequality. This yields a simple linear form for the decision rule

$$y - \frac{n}{2} > -gx$$

Or

$$\frac{y}{n} - \frac{1}{2} > -\frac{g}{n}x$$

Where

$$g = \frac{\mu/v}{\ln\left(\frac{p}{1-p}\right)}$$